A concentrated mass, W_m, traveling at a constant velocity, V_m, impacts the free end of a uniform steel rod, diameter = 1".

Find maximum compressive stress due to impact.

$W_m = 50$ lb $\quad V_m = 40$ in/sec

Rod data $\quad E = 30 \times 10^6$ psi $\quad \omega = 0.283$ lb/in^3

$L = 24$ in $\quad d = 1$" diameter

Beam Weight $\quad W_b = \frac{\pi}{4} d^2 L \omega = \frac{\pi}{4} 1^2 (24)\text{ in}^3 (.283\text{ lb/in}^3)$

$$W_b = 5.334 \text{ lb}$$

Compute Impact Force, $F_i = V_m \sqrt{\frac{k W_b}{g}}$

Rod Spring Rate $k = \frac{A_b E}{L}$

$$k = \frac{(\pi/4 1^2)\text{in}^2 (30 \times 10^6 \text{ lb/in}^2)}{24 \text{ in}} = 9.817 \times 10^5 \text{ lb/in}$$

$$F_i = 40 \frac{\text{in}}{\text{sec}} \sqrt{\frac{(9.817 \times 10^5 \text{ lb/in}) \, 5.334 \text{ lb}}{386.4 \text{ in/sec}^2}} = 4{,}656.6 \text{ lb}$$

Let the suddenly applied impact force be uniformly distributed over the rod cross sectional area. Compute uniform compressive wave stress...

$$\sigma_0 = \frac{F_i}{A_b} = \frac{4{,}656.6 \text{ lb}}{\pi/4 \, 1^2 \text{ in}^2} = 5{,}929 \text{ lb/in}^2$$

Compression

Compute reference velocity of propagation for compression wave, C.

$$C = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{30\times10^6\ lb/in^2}{(.283\ lb/in^3)/(386.4\ in/sec^2)}}$$

$$C = 202,389\ in/sec$$

The exact solution from solving differential equations to compute maximum compressive stress due to impact.

$$\sigma_{max} = \sigma_0\left(1+\sqrt{\frac{W_m}{W_b}+\frac{2}{3}}\right)$$

$$= 5,929\ lb/in^2\left(1+\sqrt{\frac{50\ lb}{5.334\ lb}+\frac{2}{3}}\right)$$

$\underline{\sigma_{max}} = \underline{24,716\ lb/in^2}$ Compression Exact Solution