# EXPERIMENTAL CORRELATION OF AN N-DIMENSIONAL LOAD TRANSDUCER AUGMENTED BY FINITE ELEMENT ANALYSIS

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## ABSTRACT

Typical designs in structural applications undergo complex loading. In order to accurately predict the behavior of complex structures accurate representation of loads is required. Traditional methods of obtaining loads involve specialized load transducers and / or modification of structures to be sensitive to specific components of load. Presented in this paper is an alternative approach to load measurement which leverages the Finite Element Method in conjunction with a physical sample to produce an n-Dimensional load transducer. Experimental verification of this method is presented along with independent approaches to load measurement. A numerical method is presented when combined with proper test methodology produces an optimal n-Dimensional load transducer.

## INTRODUCTION

Finite Element Analysis (FEA) is widely used in the design of product. While the Finite Element Analysis methodology has become much more sophisticated in recent years giving analysts greater fidelity in their models in terms of geometric representation, the analyst still needs to provide the model with loading information. Many aspects of loading can be accurately calculated. For example, in the domain of powertrain structural analysis, much of the loading can be derived from first principles such as shaking forces, gas pressure loading, bearing loads, torque, valve harmonics, etc. While these may be very complex loading scenarios, they are well defined. Deterministic mechanistic domains may also be able to calculate loading to a high level of certainty by employing multi-body dynamics calculations. There still, however, remains a large domain of structural analysis in which loading comes from difficult if not impossible to quantify external sources. Domains typical of this complex loading phenomena include vehicle (on/off road, aerospace, aquatic) loads from roads, the environment and dynamic operation of vehicle (structure). Other structures such as building and bridges also undergo complex loading which typically is difficult to quantify.

Engineers will typically instrument prototype structures with strain gauges and other transducers and compare the results of these measurements to the FEA models. Inevitably, the measured responses are due to the complex loading of the real operation of the device, whereas the FEA model typically has idealized loading scenarios. The act of attempting to correlate the FEA model is time consuming with the end result being a partial match to a subset of the experimental measurements. This inevitably results in two undesired outcomes. The first is that the engineer will make design decisions based on incomplete understanding of the loading environment. The second outcome is that extra hardware iteration loops will be required because of the incomplete knowledge of loading on the structure.

Engineers will often apply special purpose load transducers to directly measure loading at specific locations. Typical devices for this type of load measurement include wheel load transducers, load washers and custom made transducers. These devices, while providing some key information pose several problems in their application. Often wheel load transducers and custom made transducers cannot withstand the loading environment that they are measuring. The techniques deployed that make these devices sensitive to loading also makes them susceptible to failure. Typically, for these devices to be load sensitive, sections in critical load paths are reduced which makes them susceptible to overload and yielding. Load washers are also commonly used, but these devices inevitably change the stack up of components in the areas of concern and can change the actual operation of the device.

Several authors [1,2,3,5] have attempted to develop approaches to having the structure become its own load transducer. The first two authors concluded that their approach required an analysis of all possible gauge locations and combinations which they deemed too time consuming. The third author proposed a k-exchange technique deployed in D-optimal design. However, this author only posed the problem in 2 dimensional space. The fifth paper [Dhingra-Hunter] expanded this technique to the general 3D linear structure. This paper will review and document experimental correlation on a complex structure undergoing complex loading based on the techniques outlined in reference [5].

## PROBLEM FORMULATION

The applicability of the approach outlined by Dhingra-Hunter requires the structure to behave linearly under the event of interest. The structure may behave non-linearly prior to or after the event of interest. The term linear in this context refers to structures whose flexure is sufficiently small (e.g. $\sin(\theta) \approx \theta$ ). In addition, and most importantly, the strain response can be thought of as being proportional to the applied load. With these conditions being satisfied, a general expression may be written representing this proportionality:

$$[\varepsilon][C]=[F]$$

*Equation [1]*

Equation [1] is a linear relationship between the applied load cases [F] in the finite element model and the resulting strains [ε] as retrieved from the finite element model at prescribed locations and orientations. The term [C] is the matrix of proportionality that needs to be determined through standard linear algebra manipulations.

The strain matrix [ε] will have dimensions of *n* loads by *m* gauges. Each row of the strain matrix will consist of the strain as measured from the finite element model to the corresponding load case. Each column of the strain matrix corresponds to a specific location and orientation on the finite element model. These locations are determined through the D-optimal approach outlined in Dhingra-Hunter. In essence, these are virtual uniaxial strain gauges.

The loading matrix [F] has dimensions of *m* loads by *m* loads. The approach outlined in Dhingra-Hunter and applied in this paper uses a diagonal representation of the [F] matrix. However, this is not a requirement for the approach. It simply makes the bookkeeping more straight forward. Specifically, the [F] matrix is represented as:

$$[F]=\begin{bmatrix} F_1 & 0 & 0 & 0 \\ 0 & F_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & F_m \end{bmatrix}$$

*Equation [2]*

Furthermore, the values for the entries in each of the elements of the [F] matrix can be thought of as scale factors for the applied load cases. When a load case is turned "on" the corresponding value in the matrix becomes 1. Conversely when the load case is turned "off" the corresponding value becomes 0. Following this convention, when diagonal terms of the F matrix are simply represented by the value of 1, each load case is being activated independently of the other load cases. This then allows the F matrix to be represented as the identity matrix [I]. This simplifies equation [1] to be of the form:

$$[\varepsilon][C]=[I]$$

*Equation [3]*

This expression will give rise to the pseudo-inverse relationship to determine the value of the [C] matrix.

$$[C]=\left[\varepsilon^{T}\varepsilon\right]^{-1}\varepsilon^{T}$$

*Equation [4]*

The [C] matrix is the load proportionality matrix. It specifically is the proportionality relationship between strains [ε] and loads [F]. The stability of [C] is dependent upon the inverse of the dot product of the strain matrix. A nearly infinite number of configurations of the [C] matrix exist, but there are a limited number of [C] matrices that behave "well". "Well" behaving [C] matrices will be relatively insensitive to signal noise and gauge placement accuracy. In order for [C] to be stable, the determinant of the strain matrix [ε] needs to be maximum. A key attribute of the strain matrix [ε] is the condition number of the strain matrix [ε]. Heuristic evidence has shown that a very stable strain matrix [ε] has a condition number of 10 or less. Acceptable strain matrices have condition numbers of 50 or less. Systems that exhibit a condition number of 50 or more should be re-examined for suitability of load cases and candidate gauge locations. Large condition numbers indicate that the system of load cases and strain gauge locations does not have sufficient linear independence.

## EXPERIMENTAL CORRELATION

In order to show experimental correlation of the application of this technique, a simple yet sufficiently complex fixture was designed to be the subject of the loading study. The fixture was designed in such a way that forces could be calculated independently of the technique presented in this paper. The fixture created was given the nickname the "hangman" fixture (Figure 1).

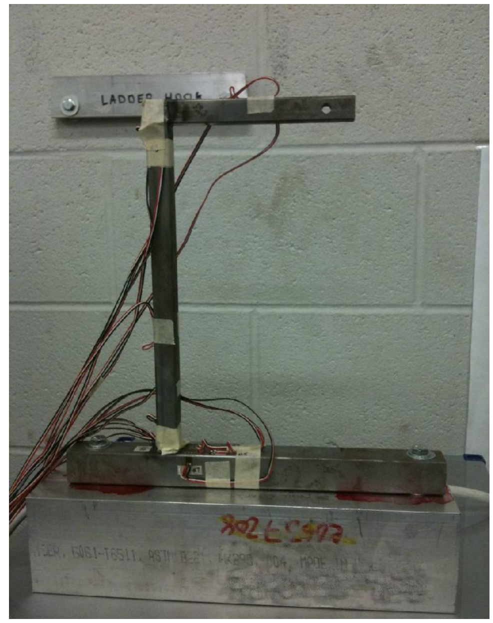

*Figure 1: Hangman Fixture*

The fixture was modeled using Abaqus/CAE Finite Element Analysis (FEA) software. The boundary conditions on the model held the fixture fixed at the two bottom most locations (Figure 2).

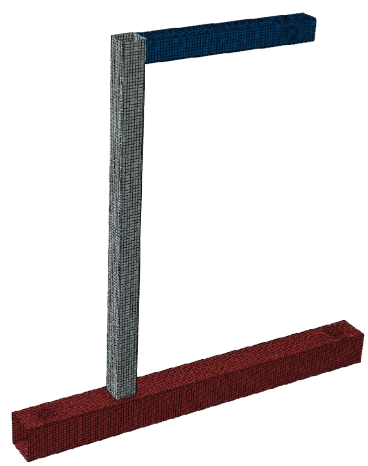

*Figure 2: Abaqus CAE model of Hangman Fixture*

Three unit load cases were chosen for the structure. These unit load cases were were nodal force loads applied at the tip of the hangman arm in the X, Y, and Z directions respectively (Figure 3).

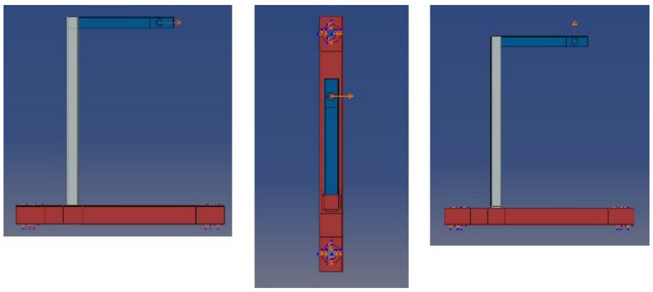

*Figure 3: Unit load cases*

These load cases were chosen to simulate the effect of environmental loading of the device being placed on the luggage rack of a motorcycle traversing a proving ground profile. The device will have a large weight attached where the loads were applied so that theoretically, any inertia loading event from the proving ground would be a linear combination of the three unit force load cases.

To locate the strain gauge locations Wolf Star Technologies True-Load software was used. This software applies the mathematical concepts outlined in this paper to locate strain gauges on the FEA model. The selection process starts with the optimal locations using a variation of the D-Optimal procedure outlined in Dhingra-Hunter[5]. The placement of the gauges are refined interactively by the analyst using the True-Load software. Typically, an analyst will modify the gauge locations to aid the technician in placing gauges aligned with reference geometry on the test article. The gauges are shown graphically in Figure 4.

These strain gauges result in a strain matrix of [ε] from the three unit load cases in the FEA model. The strains collected from the FEA model in turn are used to calculate the load proportionality matrix [C] via equation [4].

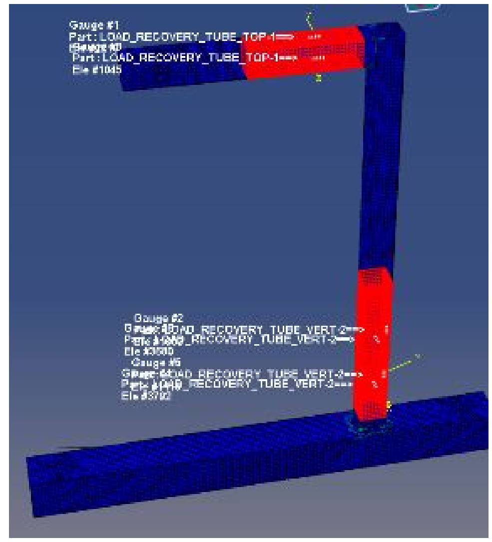


*Figure 4: Virtual Gauge Representation*

The test article was instrumented at the locations specified by True-Load (Figure 5). Two different events were chosen for the test article. The first was simple static test with dead weights on the end of the hangman fixture. The second was the proving ground dynamic event.

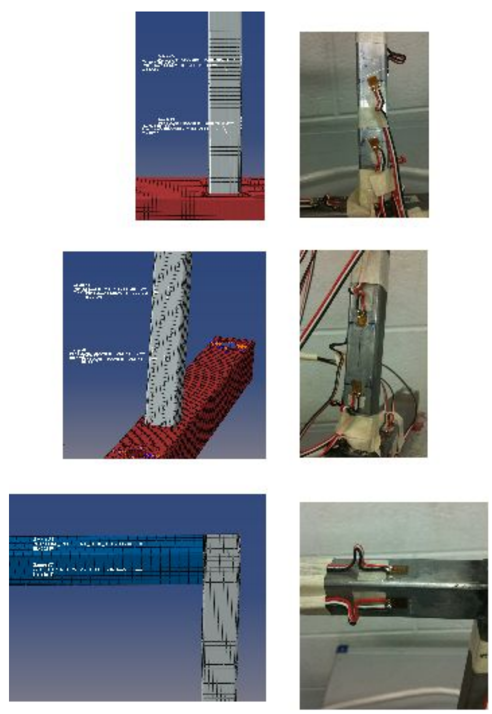

*Figure 5: Physical Strain Gauge Placement*

Typical strain readings from the 6 chosen strain locations are shown in Figure 6. Note that some of the strain gauge readings are below 20 microstrain and have significant noise to signal ratio. The static / constant load testing was performed in a variety of loading orientations. Figure 6 shows the strain results from Z oriented loading.

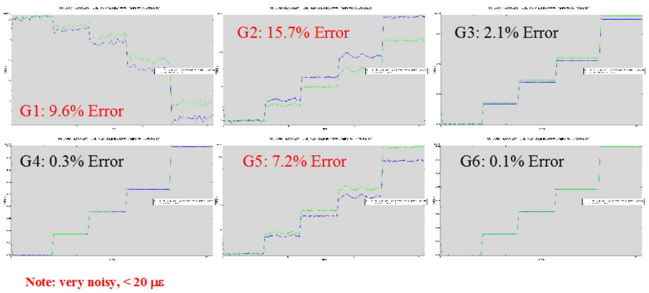


*Figure 6: Experimental & Analytical Strain Signals*

Once these strain time histories are run through True-Load/Post-Test, the resulting load cases are created by multiplying the strain signals by the load proportionality matrix [C] (Figure 6). The True-Load software calculates the simulated FEA strain based on the loading time histories. The load time histories are plotted in comparison to the actual applied dead weights. As can be seen from Figure 6, the error in load calculation is less than 2%. The measured strain and the simulated strain at gauge locations are plotted in Figure 8 along with the RMS error. As is shown in these plots, the error in simulated strain is less than 1% for the gauges with strains over 20 microstrain.

For the dynamic proving ground load cases, accelerometers were mounted on the "hangman" mass. The entire "hangman" fixture was mounted to the luggage rack of a test motorcycle (Figure 7).

The calculated force was a simple multiplication of the "hangman" mass times the measured acceleration. Figure 8 shows the forces calculated from the accelerometers plotted over the forces calculated using the strain measurements. Again the force calculations show that there is an error less than 2% except for the case of the vertical loading. In the case of vertical loading, the mass used for the force calculation is missing the distributed mass of the structure. Even with this error, the difference is approximately 6%. The forces coming from the strain gauge measurements are probably more accurate since the load calculation from strain measurement takes into account the distributed flexibility of the entire structure.

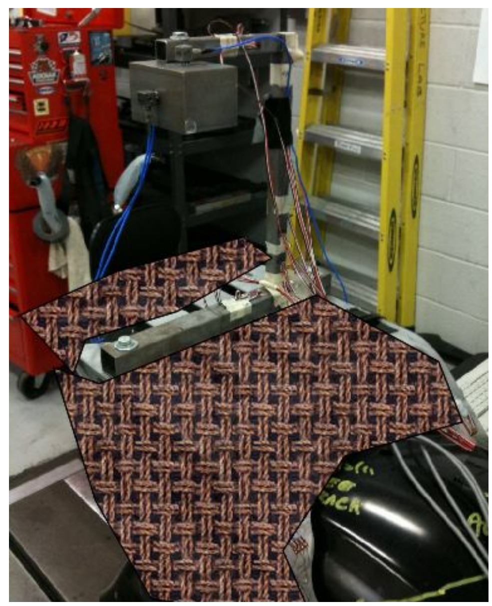

*Figure 7: "Hangman" Fixture mounted on Test Vehicle*

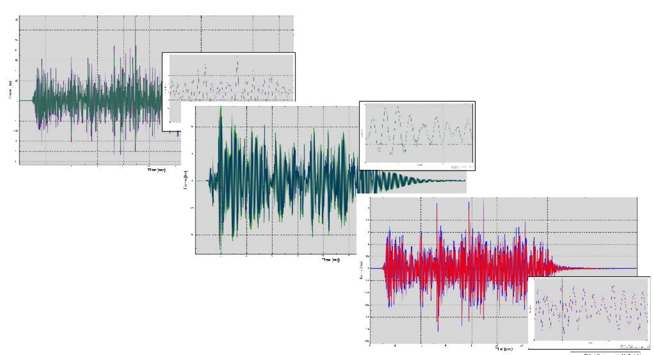

*Figure 8: Dynamic Force Correlation Plot*

Figure 9 shows the measured and simulated strains which again show that the error is less than 1% across all of the strain channels.

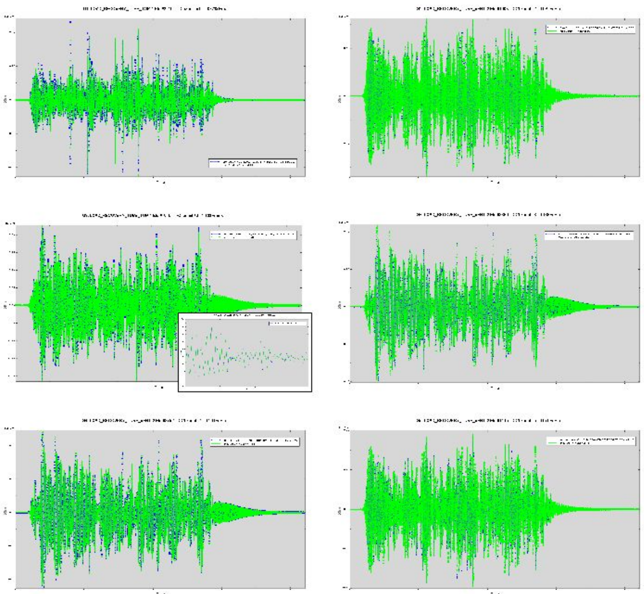

*Figure 9: Dynamic Experimental and Analytic Strain Correlation Plot*

## CONCLUSIONS

The experimental data illustrate the technique of linking the physical test to the FEA model through use of a strain proportionality matrix [C] is accurate, efficient and effective. The technique illustrates that any structure which operates linearly in a test environment can become its own load transducer through proper management of FEA data and strain data collection.

This method may be extended into the domain of linear dynamics [6]. Instead of using unit load cases, the mode shapes would provide the analytical strain matrix [$\varepsilon$]. The load scaling functions that are calculated would in essence be modal participation functions.

With relatively simple and efficient means, an analyst can quickly capture the complete linear quasi-static and dynamic response of any structure. The Wolf Star Technologies True-Load© software combined with Dassault Systems Abaqus/CAE makes an easy and efficient environment for applying this approach. Through the application of this technique, the analyst makes better design decisions through thorough understanding of the loading environment. With better understanding and better design decisions being made, fewer learning cycles in hardware will need to be performed. This ultimately leads to faster, more efficient product development.

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